Strategies to Manage Errors in Online Mathematics Tutoring

By: Jun Xie and Xiangen Hu, University of Memphis

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Introduction

Educators and researchers usually consider errors to be favorable to learning (Bjork, Dunlosky, & Kornell, 2013; Schwartz & Bransford, 1998; VanLehn, Siler, & Murray, 2003). This opinion might originate from behaviorism, which contends that learning is a gradual process of trial and error. For example, behaviorism researchers found that after a series of failed attempts, rats learned the structure of a maze and found the correct path to the destination (Olton & Samuelson, 1976).

Studies revealed that errors might slow humans’ learning process but that they have a positive effect on long-term knowledge retention and performance on tests of similar content (Lee, 2012; Simon & Bjork, 2001; Taylor & Rohrer, 2010). Errors can cause learners to pay attention and motivate metacognitive activities to diagnose mistakes, modify learning strategies, and monitor the effectiveness of learning (Keith & Frese, 2005). After systematically reviewing studies on learning, Bjork, Dunlosky, and Kornell (2013) concluded that errors should be treated as an essential component of effective learning instead of a representation of learners’ inadequacies.

Because errors are so crucial in learning, the question arises of how learners manage errors so as to learn from them. Do learners ask for help to avoid errors or insist on solving problems on their own until they master the learning content? Do they ask for help and solve problems with assistance? What factors influence learners’ choices of strategies to manage errors? Bjork and colleagues (2013) postulated that managing errors was important for mastering knowledge. Few studies have been done on students’ strategies to manage errors. This paper reports on a study of error-management strategies and thus provides more evidence on how students regulate their learning behaviors to learn from errors. In addition, the study may benefit developers or intelligent tutoring systems (ITS) in designing instruction that optimizes ITS’s positive effect on learning.

The objective of this study was to examine learners’ strategies to manage errors, the factors that influenced the management of errors, and the relationship between strategies and learning outcomes in Assessment and Learning in Knowledge Spaces (ALEKS). ALEKS is an adaptive ITS that provides the student the topics that he or she is most ready to learn. ALEKS uses worked examples as explanations to the problems. This study adopted the framework of evidence-centered design (ECD) to investigate the error-management strategies because the strategies are the complex products of the interaction between the learning environment and individuals. ECD provides an effective framework for measuring constructs that are hard to capture in a digital environment.
Student Model

For this study, we defined three categories of strategies for managing errors: help-seeking, self-correction, and a mix of help-seeking and self-correction. We postulated that these three strategies might occur in different learning phases. VanLehn (1996) proposed that there are three phases during learning: early, intermediate, and late. In the early phase, students need instructional materials to gain basic understanding of the domain. Then as their knowledge grows, students might devote more attention to problem solving—the intermediate phase. In the late phase, students might focus on problem solving to improve their ability and speed in applying knowledge. Therefore, in the early phase, students might make errors because they lack knowledge. They might ask for help after making errors more frequently than in the other two phases. In the intermediate phase, students’ domain knowledge is greater, which might enable them to attempt to correct their errors without seeking help. In the late phase, students have enough domain knowledge to correct errors by themselves, and most of the errors might be caused by carelessness.

Past studies also found that prior knowledge influences students’ strategies to manage errors. Wood & Wood (1999) reported that students with high prior knowledge preferred to self-correct whereas students with low prior knowledge tended to overuse help. Students with high prior knowledge relied more on the schemas stored in their long-term memory to solve problems (Kalyuga, 2007). Thus, students with higher prior knowledge might be more likely to adopt self-correction to manage errors. Self-regulated-learning models theorize that prior knowledge is positively related to the use of cognitive and metacognitive strategies across situations and contexts (Vanderstoep, Pintrich, & Fagerlin, 1996). The theory is that students with high prior knowledge have better self-regulation so they are more organized when using error-management strategies. Similarly, when students make careless errors, this might indicate that they have high prior knowledge and that they may be more likely to apply more organized error-correction patterns.

Strategies to manage errors vary in their effects on learning outcomes. Renkl and Atkinson (2003) found that self-correction facilitated students’ mastery of skills in the late learning phase. Roll, Baker, Aleven, and Koedinger (2014) found that help-seeking on challenge steps fostered learning whereas overusing help was associated with poor learning. Researchers also reported that the overuse of worked examples prevented students from improving learning outcomes (Renkl & Atkinson, 2003). Therefore, in online tutoring systems based on worked examples, using excessive help to manage errors might have negative effects on students’ learning. Self-correction or a mix of help and self-correction might benefit students in mastering skills. Furthermore, the existing studies (Snow, Allen, Javocina, & McNamara, 2015; San Pedro, Snow, Baker, McNamara, & Heffernan, 2015) indicated that the randomness of students’ behaviors affected learning outcomes. Snow, Allen, Javocina, and McNamara (2015) found that the more
organized the pattern of students’ choices in the iSTART-2 system, the higher the self-explanation quality. Therefore, organized behavior during learning might generate better learning outcomes. The effect of strategies to manage errors may be influenced by prior knowledge. Students with high prior knowledge benefited from self-correction (Wood & Wood, 1999). They did not benefit as much as those with low prior knowledge from the explanations based on worked examples (Kalyuga, 2007; Renkl & Atkinson, 2003). Thus, in an ITS based on worked examples, self-correction may favor the performance of students with high prior knowledge, but for those with low prior knowledge help-seeking positively associated with posttest.

Task Model

Task Environment: ALEKS

The study investigated students’ strategies to manage errors in Assessment and LEarning in Knowledge Spaces. ALEKS is an adaptive tutoring system built on knowledge space theory. ALEKS is distinct from other similar math tutors (e.g., Cognitive Tutor). First, students solve the problem fitting their knowledge state. A knowledge state is defined as a specific subset of problems that a student is able to solve (Falmagne, Koppen, Villano, Doignon, & Johannesen, 1990). Knowledge space theory determines what the student is ready to learn based on his or her knowledge state. Second, each topic has enough instances of problems to ensure that the student obtains enough practice. If the student does not solve a problem successfully, he or she cannot repeat the same problem but can access another instance in the next step. ALEKS requires the student to input each problem-solving step. Third, each explanation in ALEKS includes the problem and all the steps of the solution that can be termed worked example. Students can read the explanation to the same problem only once.

Task Description

During an afterschool program, sixth-grade students learned math topics in ALEKS 2 days a week. Each program day lasted for 2 hours, and there were 50 days in total. Students took rests every 20 minutes in a program day. ALEKS allowed students to choose to read explanations or solve instances of problems on their own after making errors so that the strategies for managing errors could be observed naturally. To ensure that students used all three strategies in ALEKS, the topic difficulty dynamically matched students' knowledge states. Difficult topics may impel students to rely on help-seeking, and easy topics may induce students to adopt only self-correction. Furthermore, to inhibit students from gaming the system (whereby students frequently make errors and then ask for help in order to get the correct answers), ALEKS did not
provide bottom-out hints. On each step, students solved different problem instances. Most questions required that students input the detailed problem-solving steps.

The ALEKS log file recorded the students’ responses on each step during learning and time spent on each step. The log file contained 118,995 error steps out of 336,842 learning steps (35.3% error steps).

**Potential Task Products**

The strategies for managing errors in this study were defined in terms of the next two steps students took after errors: *help-seeking strategy*, students only read explanations in the next two steps after errors; *self-correction strategy*, students only solved problem instances in the next two steps after errors; and *mixed strategy*, students not only read explanations, but also solved another problem instance in the next two steps after errors.

In addition, because students selected the strategy on their own, the randomness of use of the strategies was observed as well.

From the ALEKS log file, the probability of slip can be captured. Students’ prior knowledge and posttest results were also collected.

**Evidence Model**

**Observations of Strategies to Manage Errors**

We used conditional probability (D’Mello, 2012) to measure the transitional likelihood from the error to a strategy (formula 2). $C$ represented the error, and $X$ represented a strategy. A probability higher than zero indicated that the strategy occurred beyond its base rate.

**Formula 2. The transitional likelihood from errors to a strategy**
*(D’Mello, 2012)*

\[
L[C \rightarrow X] = \frac{Pr[X \cap C]}{Pr[C]} - \frac{Pr[X]}{1 - Pr[X]}
\]
Where

\[ L[C\rightarrow X]: \text{the likelihood that a student transfers from an error to a strategy} \]
\[ \Pr[X\cap C]: \text{the probability that a strategy occurred after errors} \]
\[ \Pr[X]: \text{the probability of strategy use during learning} \]
\[ \Pr[C]: \text{the probability of errors during learning}. \]

This study applied Shannon entropy (Shannon, 1951) to the observations of strategies. Shannon entropy is usually used to measure disorder or uncertainty, but it has also been applied to capture dynamic changes in students’ choices (Snow, Allen, Javocina, & McNamara, 2015; San Pedro, Snow, Baker, McNamara, & Heffernan, 2015) and learning phase transitions before insight (Stephen, Boncoddo, Magnuson, & Dixon, 2009). A high entropy value indicated that a student’s choices of strategies were disorganized, whereas a low entropy indicated that a student’s choices of strategies were organized (formula 1).

\[ H(X) = -\sum_{i=1}^{n} p(x_i) \log p(x_i) \]

The contextual slip model was used to calculated the probability of a slip (Baker, Corbett, & Aleven, 2008a, 2008b). A higher probability of slip meant that the error was most likely caused by carelessness.

To capture prior knowledge, students’ fifth-grade scores on the Tennessee Comprehensive Assessment Program (TCAP) were collected. The students whose fifth-grade TCAP scores were higher than the medium score (44) were clustered in a high prior knowledge group \((M = 29.72, SD = 12.86)\). Other students were grouped as low prior knowledge \((M = 58.32, SD = 9.66)\). The posttest was measured by students’ sixth-grade TCAP score.

**Likelihood of Strategy Use**

One-way Anova analysis revealed significant differences in likelihood of the three strategies’ transitions from errors, \(F(2,654) = 731.06, p < .000, \eta^2 = .69\). The multiple comparisons showed that the likelihood of help-seeking \((M = .04, SD = .04)\) was significantly lower than that of self-correction \((M = .31, SD = .11)\) or the mixed strategy \((M = .33, SD = .11)\). There was no difference between the likelihoods of self-correction and the mixed strategy. Self-correction and the mixed strategy were most frequently used by students.
Randomness of Strategies

Pearson correlation indicated that the randomness of strategies was negatively associated with prior knowledge, $r(219) = -0.16, p < .025$. The students with lower prior knowledge displayed a more disorganized choice of strategies. The randomness of strategies also was negatively linked to the probability of slip, $r(219) = -0.19, p < .025$. A disorganized pattern tended to relate to a lower probability of slip, meaning that students tended to make errors because of a lack of knowledge. This implies that students with less knowledge in the domain might display a more disorganized pattern of choosing strategies to manage errors. Hence, the relationship we found between randomness and prior knowledge was consistent with the finding of a relationship between randomness and slip: Students with low prior knowledge were inclined to show disorganized patterns of choosing strategies to manage errors.

Relationships of strategies with prior knowledge and slip

The Pearson correlations implied that randomness had a positive relationship with the likelihood of help-seeking and the mixed strategy but was negatively associated with the likelihood of self-correction. That is, when students more frequently used help-seeking or the mixed strategy, they tended to be disorganized in choosing strategies. When students used self-correction more, their strategy choice was more organized (Table 1). The likelihood of use of the three strategies did not relate to prior knowledge. The likelihood of self-correction, however, was positively linked to the probability of slip.

Table 1. Pearson correlations of likelihood of strategies with prior knowledge, the probability of slip, and randomness

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Randomness</th>
<th>Prior knowledge</th>
<th>Probability of slip</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r$</td>
<td>$p$</td>
<td>$r$</td>
</tr>
<tr>
<td>Help-seeking</td>
<td>.69$^*$</td>
<td>.000</td>
<td>-.79</td>
</tr>
<tr>
<td>Self-correction</td>
<td>-.43$^{**}$</td>
<td>.000</td>
<td>.16</td>
</tr>
<tr>
<td>Mixed</td>
<td>.19$^*$</td>
<td>.005</td>
<td>.08</td>
</tr>
</tbody>
</table>

$^*$ $p < .005, ^{**} p < .000$

Relationships Between Strategies and Posttest

The regression results suggested that the likelihood of use of the three strategies was positively associated with posttest scores whereas the randomness of the strategies was negatively linked to posttest scores. The three strategies can significantly explain 11.1% variance in the posttest scores, $F(4,214) = 6.69, p < .000$. 

A4L Analytics for Learning
For students with low prior knowledge, the likelihood of strategy use and randomness did not relate to their posttest results. For students with high prior knowledge, the likelihood of self-correction and mixed strategies positively related to their posttest scores. The three strategies accounted for a 17% variance in posttest scores in the high prior knowledge group. Table 2 presents the detailed regression results.

Table 2. Regressions of the three strategies on posttest

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Randomness</th>
<th>Prior knowledge</th>
<th>Probability of slip</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>p</td>
<td>B</td>
</tr>
<tr>
<td>Intercept</td>
<td>39.12</td>
<td>.053</td>
<td>56.78</td>
</tr>
<tr>
<td>Likelihood of help-seeking</td>
<td>83.38**</td>
<td>.028</td>
<td>26.17</td>
</tr>
<tr>
<td>Likelihood of self-correction</td>
<td>63.29**</td>
<td>.000</td>
<td>.31</td>
</tr>
<tr>
<td>Likelihood of mixed</td>
<td>58.17**</td>
<td>.000</td>
<td>-3.89</td>
</tr>
<tr>
<td>Randomness</td>
<td>-33.89*</td>
<td>.024</td>
<td>-23.04</td>
</tr>
</tbody>
</table>

*p < .05.  **p < .00.

Discussion

The study defined the strategies and captured the factors influencing their use. The findings indicated that students most frequently used self-correction and the mixed strategy to manage errors. When making careless errors or displaying organized behavior patterns, students were more likely to choose self-correction to manage errors. When displaying disorganized behavior patterns, students were more likely to select help-seeking to manage errors. In addition, higher prior knowledge and careless errors were positively related to an organized pattern of strategy choice. Help-seeking, self-correction, and the mixed strategy were positively associated with posttest scores. For students with high prior knowledge, self-correction and the mixed strategy boosted their posttest results.

This study found that the probability of slip positively related to organized patterns of strategy choice and self-correction. One way to interpret this is that errors with a higher probability of slip might occur during the intermediate and late learning phases. In those phases, students are developing organized patterns as they acquire knowledge and are more focused on problem solving. Thus, the greater probability of slip might indicate students’ positive progress in learning.

The reason that disorganized strategy-choice patterns were positively linked to help-seeking might be that excessive help-seeking implied that students were in the early learning phase. In that phase, students
lack prior knowledge in the domain, so they are exploring effective learning strategies. Thus, their choice of strategies appear more disorganized.

The findings on the relationship of prior knowledge and slip with randomness were consistent with self-regulated–learning theory (Vanderstoep et al., 1996). On the basis of the findings, students with low prior knowledge might not have developed stable patterns for learning and might still have been exploring appropriate learning strategies. The learning system therefore might provide more instructions for them to help form organized learning patterns.

That help-seeking, self-correction, and the mixed strategy all benefited posttest scores might indicate that most of the students made the right choice of strategies in different learning phases, so that all the strategies are positively related to learning. For students with high prior knowledge, the relationship between strategies and posttest scores generally confirmed the results of the Aleven and Koedinger study (2001).

Future studies need to use fine-grained method to investigate the patterns of use of the strategies during the course of learning so that the findings of this study can be interpreted more accurately.
References


Appendix: Design Pattern

<table>
<thead>
<tr>
<th>Authors</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Name</td>
</tr>
<tr>
<td>Last Name</td>
</tr>
<tr>
<td>Affiliation</td>
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<tr>
<td>E-Mail</td>
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<tr>
<td>First Name</td>
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<tr>
<td>Last Name</td>
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<tr>
<td>Affiliation</td>
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<td>E-Mail</td>
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<table>
<thead>
<tr>
<th>Overview</th>
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<tbody>
<tr>
<td>Summary</td>
</tr>
<tr>
<td>• The strategies to manage errors refers to how students learn from errors by self-regulating their learning behaviors. During learning in an intelligent tutoring system (ITS), students are able to choose help-seeking or self-correction after making errors, or they use both help-seeking and self-correction to learn from errors.</td>
</tr>
<tr>
<td>• Students learned sixth-grade math topics in Assessment and LEarning in Knowledge Spaces (ALEKS). ALEKS is an adaptive ITS that can match topics’ difficulty with students’ knowledge states. Therefore, it determines the topics that students are most ready to learn.</td>
</tr>
<tr>
<td>• The data included log data in ALEKS (e.g., help-seeking and errors) and individual differences.</td>
</tr>
</tbody>
</table>


**Rationale**

- Educators and researchers usually consider errors to be favorable to learning.
  - Learning is a gradual process of trial and error.
  - Errors have a positive effect on learners’ long-term knowledge retention and performance on tests of similar content.
  - Errors can cause learners to pay attention and motivate metacognitive activities to diagnose mistakes, modify learning strategies, and monitor the effectiveness of learning.
- Errors should be treated as an essential component of effective learning instead of a representation of learners’ inadequacies.
- Few studies have been done on students’ strategies to manage errors. A study on strategies to manage errors can provide more evidence on how students regulate their learning behaviors to learn from errors.
- The study on error-management strategies may benefit intelligent tutoring system (ITS) developers in designing instruction that optimizes ITS’s positive effect on learning.

**Student Model**

<table>
<thead>
<tr>
<th>Focal Construct</th>
<th>The strategies to manage errors referred to the student’s learning behaviors after the error including help-seeking, self-correction, and mixed.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>- Help-seeking: the student asked for explanations in the subsequent two steps.</td>
</tr>
<tr>
<td></td>
<td>- Self-correction: the student continued to solve problems in the subsequent two steps.</td>
</tr>
<tr>
<td></td>
<td>- Mixed: the student asked for one explanation and solved one problem in the subsequent two steps.</td>
</tr>
</tbody>
</table>

| Additional knowledge, skills, and abilities | Prior knowledge, the probability of slip, and posttest scores may relate to students’ strategies to manage errors. |

**Task Model**

<table>
<thead>
<tr>
<th>Characteristic Features of the Task</th>
<th>ALEKS allowed students to choose to read explanations or solve similar problem instances on their own after making errors so that the strategies for managing errors could be observed naturally.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>To ensure that students used all three strategies in ALEKS, the topic difficulty dynamically matched students’ knowledge states. Difficult topics may impel students to rely on help-seeking, and easy topics may induce students to adopt only self-correction.</td>
</tr>
<tr>
<td></td>
<td>To inhibit students from gaming the system (whereby students frequently make errors and then ask for help in order to get the correct answers), ALEKS did not provide bottom-out hints. On each step, students solve different problem instances.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable Features of the Task</th>
<th>Students’ choices on each step (e.g. ask for the explanation, and solve problem)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time spent on each step</td>
</tr>
<tr>
<td></td>
<td>ALEKS’s feedbacks on students’ solutions (e.g. the answer is correct or incorrect)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Potential Task Products</th>
<th>The likelihood of strategies to manage errors occurring after errors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>- <strong>Help-seeking</strong>: student only read explanations in the next two steps after errors.</td>
</tr>
<tr>
<td></td>
<td>- <strong>Self-correction</strong>: students only solved problem instances in the next two steps after errors.</td>
</tr>
<tr>
<td></td>
<td>- <strong>Mixed</strong>: students not only read explanations but also solved another problem instance in the next two steps after errors.</td>
</tr>
<tr>
<td></td>
<td>The randomness of strategy choice (measured by Shannon entropy)</td>
</tr>
<tr>
<td></td>
<td>Individual differences</td>
</tr>
<tr>
<td></td>
<td>- Prior knowledge</td>
</tr>
<tr>
<td></td>
<td>- Posttest scores</td>
</tr>
<tr>
<td></td>
<td>- The probability of slip</td>
</tr>
</tbody>
</table>
Evidence Model

Potential Observations

- The likelihood of strategy use to manage errors
  - Self-correction and mixed strategy significantly different from help-seeking
  - Self-correction positively linked to probability of slip
  - Self-correction negatively associated with randomness
  - Help-seeking and mixed strategy positively related to randomness

- Randomness
  - Negatively associated to prior knowledge
  - Negatively linked to the probability of slip
  - Negatively linked to posttest scores

- The relationships between strategies and posttest
  - The likelihood of use of the three strategies positively related to posttest scores
  - For students with high prior knowledge, self-correction and mixed strategy positively associated with posttest scores

Potential Frameworks

- When the topic difficulty matched students’ knowledge states dynamically, students more frequently selected self-correction and the mixed strategy to learn from errors.
- When student’s displayed more disorganized patterns of strategy choice, they probably more frequently used help-seeking or the mixed strategy. Otherwise, they more frequently used self-correction.
- Students with higher knowledge in the domain tended to show more organized patterns of strategy choices and may use more self-correction.
- The three strategies favored students’ posttest scores in the adaptive learning environment.
- For students with high prior knowledge, the strategy of self-correction and the mixed strategy boosted their posttest scores.